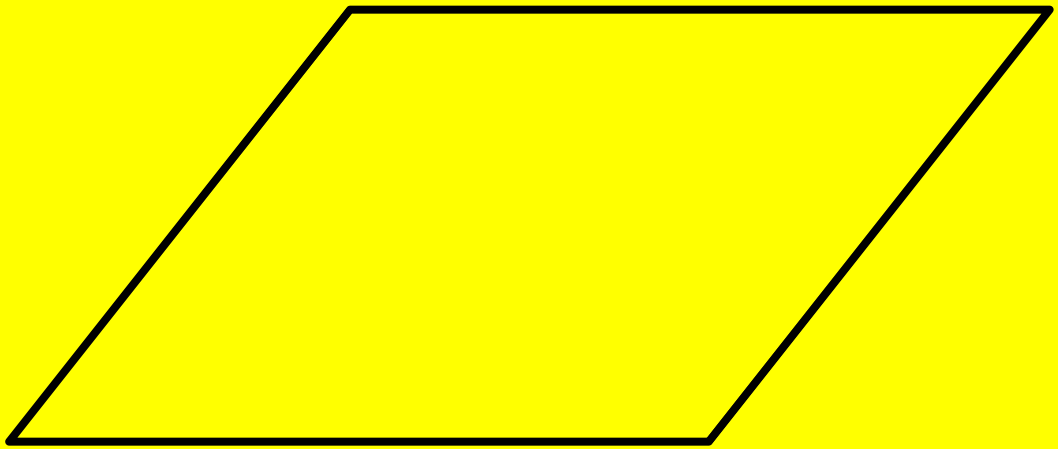
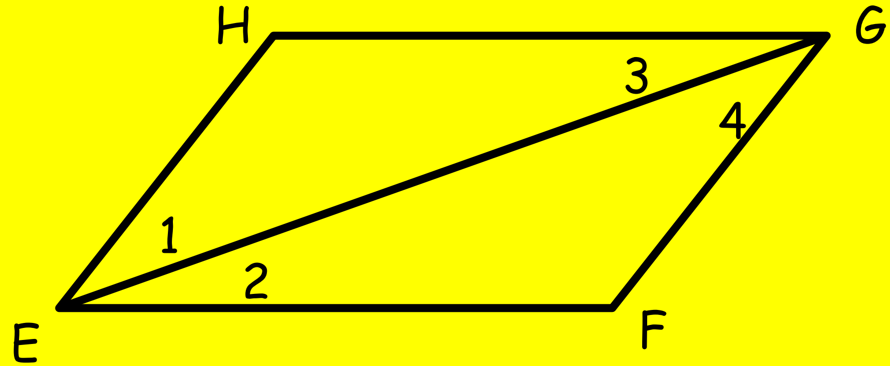
Definition: A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.



There are several properties of parallelograms in the form of theorems. We will look at these properties and their converses today.

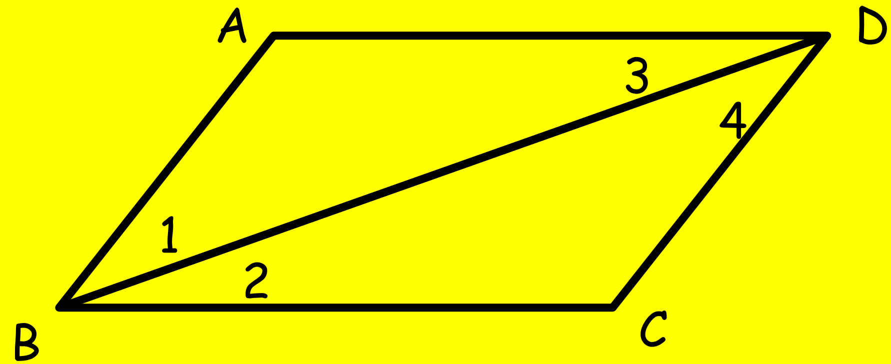
Theorem 5-1: <u>Opposite sides of a parallelogram are congruent.</u>



1. HGEF is a parallelogram	1. Given
	3. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
6. $\overline{HG} \cong \overline{EF}$; $\overline{HE} \cong \overline{GF}$	

Converse (Theorem 5-4): <u>If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.</u>

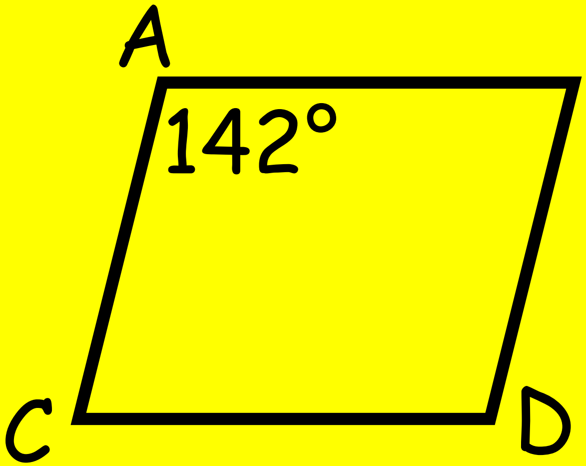
Theorem 5-2: <u>Opposite angles of a parallelogram are congruent.</u>



1. ABCD is a parallelogram	1. Given
6. $\angle A \cong \angle C$	

Converse (Theorem 5-6): <u>If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.</u>

Example 1:



m∠B = ___38___

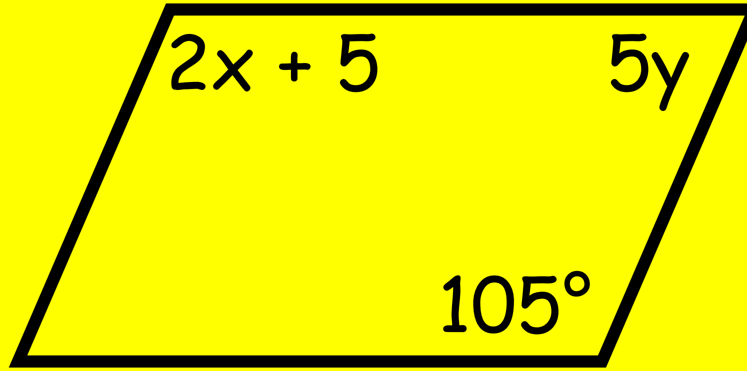
m∠C = ___38___

m∠D = ___142°___

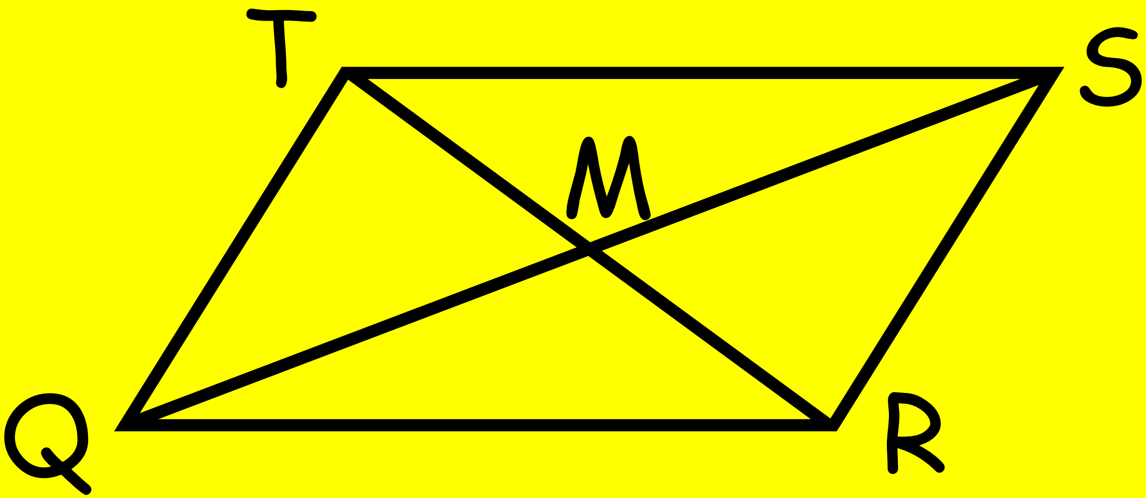
m∠A + m∠B = ___180___

m∠A + m∠B + m∠C + m∠D = ___360___

Example 2:



Theorem 5-3: Diagonals of a parallelogram bisect one another.

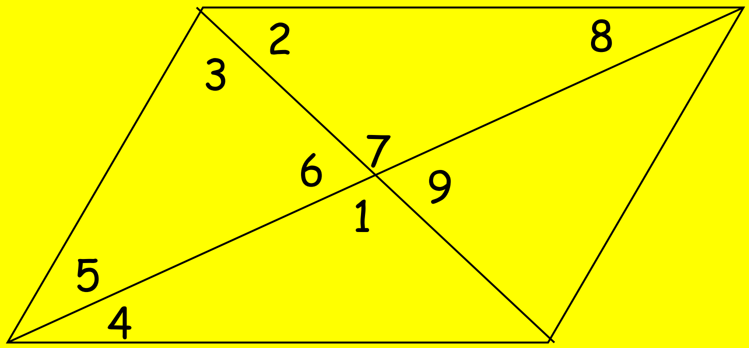


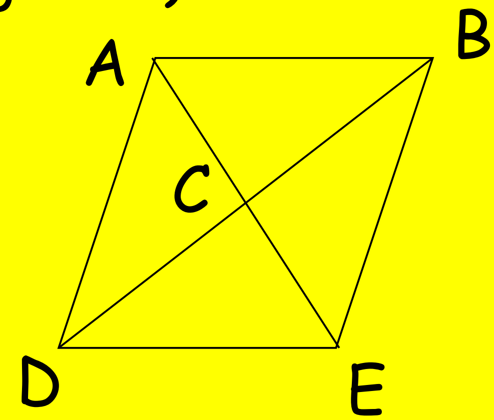
M is the midpoint of $\overline{TR}$ and $\overline{QS}$

Therefore, TM = MR and QM = MS.

Converse (Theorem 5-7): If diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.

Examples (all figures are parallelograms):





$AC = 3x - 1$

$CE = \frac{1}{2}x + 9$

$CD = \frac{3}{4}y$

$CB = y - 4$

$m\angle 1$ = ______ $m\angle 2$ = ______

$m\angle 3 = 80$ $m\angle 4$ = ______

$m\angle 5$ = ______ $m\angle 6 = 80$

$m\angle 7$ = ______ $m\angle 8 = 40$

$m\angle 9$ = ______

x = _____ AC = _____

CE = _____ AE = _____

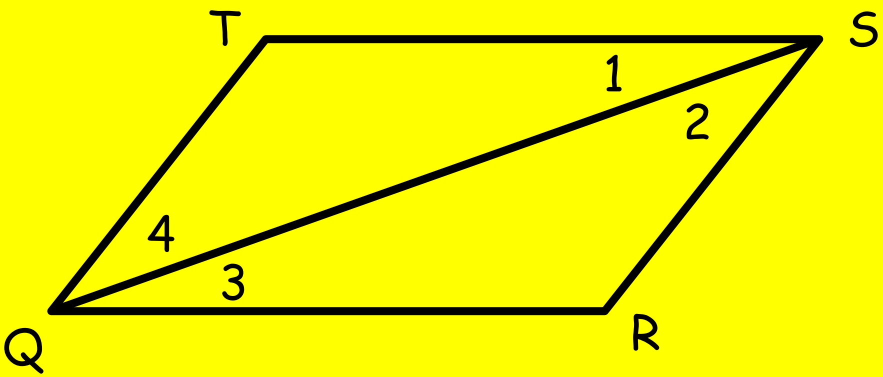
y = _____ CD = _____

CB = _____ DB = _____

Theorem 5-5: If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

Given: $\overline{TS} \cong \overline{QR}$; $\overleftrightarrow{TS} \parallel \overleftrightarrow{QR}$

Prove: TSRQ is a parallelogram.



1. $\overline{TS} \cong \overline{RQ}$; $\overleftrightarrow{TS} \parallel \overleftrightarrow{RQ}$	1. Given
2. $\angle 1 \cong \angle 3$	2. If two parallel lines are cut by a transversal, then alternate interior angles are congruent
5. $\angle 4 \cong \angle 2$	
	6. If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.
7. TSRQ is a parallelogram	7. Definition of a parallelogram.

RECAP: There are 5 ways to prove that a quadrilateral is a parallelogram. They are…

1. If both pairs of opposite sides are parallel, then the quadrilateral is a parallelogram. (Definition of a Parallelogram).

2. If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.

3. If one pair of opposite sides are congruent AND parallel, then the quadrilateral is a parallelogram.

4. If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.

5. If the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.